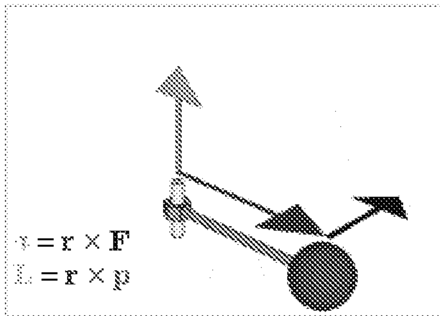
Torque

From Wikipedia, the free encyclopedia



Relationship between force **F**, torque **τ**, linear momentum **p**, and angular momentum **L** in a system which has rotation constrained in one plane only. (Forces and moments due to gravity and friction not considered.)

Torque, also called **moment** or **moment of force** (see the terminology below), is the tendency of a force to rotate an object about an axis,[1] fulcrum, or pivot. Just as a force is a push or a pull, a torque can be thought of as a twist.

In more basic terms, torque measures how hard something is rotated. For example, imagine a wrench or spanner trying to twist a nut or bolt. The amount of "twist" (torque) depends on how long the wrench is, how hard you push on it, and how well you are pushing it in the correct direction.

The terminology for this concept is not straightforward: In physics, it is usually called "torque", and in mechanical engineering, it is called "moment".[2] However, in mechanical engineering, the term "torque" means something *different*,[3] described below. In this article, the word "torque" is always used in the physics sense, synonymous with "moment" in engineering.

The symbol for torque is typically τ, the Greek letter *tau*. When it is called moment, it is commonly denoted M.

The magnitude of torque depends on three quantities: First, the force applied; second, the length of the *lever arm*[4] connecting the axis to the point of force application; and third, the angle between the two. In symbols:

$$\boldsymbol{\tau} = \mathbf{r} \times \mathbf{F}$$
$$\tau = rF \sin\theta$$

where

$\boldsymbol{\tau}$ is the torque vector and τ is the magnitude of the torque,
r is the displacement vector (a vector from the point from which torque is measured to the point where force is applied), and r is the length (or magnitude) of the lever arm vector,
F is the force vector, and F is the magnitude of the force,
$\times$ denotes the cross product,
θ is the angle between the force vector and the lever arm vector.

The length of the lever arm is particularly important; choosing this length appropriately lies behind the operation of levers, pulleys, gears, and most other simple machines involving a mechanical advantage.

The SI unit for torque is the newton meter (N·m). In Imperial and U.S. customary units, it is measured in foot pounds (ft·lbf) (also known as 'pound feet') and for smaller measurement of torque: inch pounds (in·lbf) or even inch ounces (in·ozf). For more on the units of torque, see below.

Contents

Terminology

See also: Couple (mechanics)

In mechanical engineering (unlike physics), the terms "torque" and "moment" are not interchangeable. "Moment" is the general term for the tendency of one or more applied forces to rotate an object about an axis (the concept which in physics is called torque).[3] "Torque" is a special case of this: If the applied force vectors add to zero (i.e., their "resultant" is zero), then the forces are called a "couple" and their moment is called a "torque".[3]

For example, a rotational force down a shaft, such as a turning screw-driver, forms a couple, so the resulting moment is called a "torque". By contrast, a lateral force on a beam produces a moment (called a bending moment), but since the net force is nonzero, this bending moment is not called a "torque".

This article follows physics terminology by calling all moments by the term "torque", whether or not they are associated with a couple.

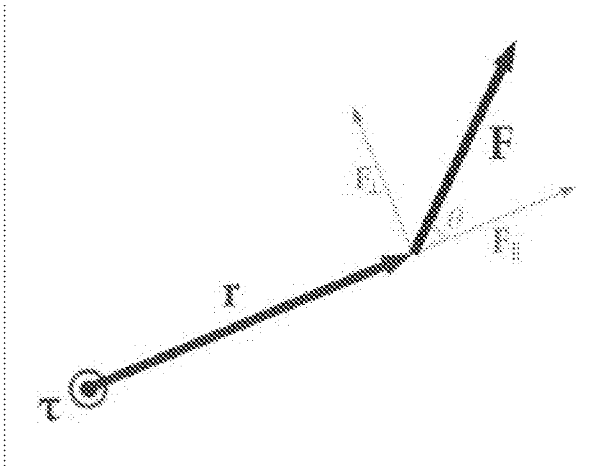
History

The concept of torque, also called moment or couple, originated with the studies of Archimedes on levers. The rotational analogues of force, mass, and acceleration are torque, moment of inertia, and angular acceleration, respectively.

Definition and relation to angular momentum

A force applied at a right angle to a lever multiplied by its distance from the lever's fulcrum (the length of the lever arm) is its torque. A force of three

newtons applied two meters from the fulcrum, for example, exerts the same torque as a force of one newton applied six meters from the fulcrum. The direction of the torque can be determined by using the right hand grip rule: if the fingers of the right hand curl in the direction of rotation and the thumb points along the axis of rotation, then the thumb also points in the direction of the torque.[5]



A particle is located at position **r** relative to its axis of rotation. When a force **F** is applied to the particle, only the perpendicular component $\mathbf{F}_\perp$ produces a torque. This torque $\boldsymbol{\tau} = \mathbf{r} \times \mathbf{F}$ has magnitude $\tau = |\mathbf{r}|\ |\mathbf{F}_\perp| = |\mathbf{r}|\ |\mathbf{F}|\ \sin\theta$ and is directed outward from the page.

More generally, the torque on a particle (which has the position **r** in some reference frame) can be defined as the cross product:

$$\boldsymbol{\tau} = \mathbf{r} \times \mathbf{F},$$

where **r** is the particle's position vector relative to the fulcrum, and **F** is the force acting on the particle. The magnitude τ of the torque is given by

$$\tau = rF\sin\theta,$$

where r is the distance from the axis of rotation to the particle, F is the magnitude of the force applied, and θ is the angle between the position and force vectors. Alternatively,

$$\tau = rF_\perp,$$

where $F_\perp$ is the amount of force directed perpendicularly to the position of the particle. Any force directed parallel to the particle's position vector does not produce a torque.[6]

It follows from the properties of the cross product that the torque vector is perpendicular to both the position and force vectors. It points along the axis of rotation, and its direction is determined by the right-hand rule.[6]

The torque on a body determines the rate of change of the body's angular momentum,

$$\boldsymbol{\tau} = \frac{\mathrm{d}\mathbf{L}}{\mathrm{d}t}$$

where **L** is the angular momentum vector and t is time. If multiple torques are acting on the body, it is instead the net torque which determines the rate of change of the angular momentum:

$$\boldsymbol{\tau}_1 + \cdots + \boldsymbol{\tau}_n = \boldsymbol{\tau}_{\mathrm{net}} = \frac{\mathrm{d}\mathbf{L}}{\mathrm{d}t}.$$

For rotation about a fixed axis,

$$\mathbf{L} = I\boldsymbol{\omega},$$

where I is the moment of inertia and $\boldsymbol{\omega}$ is the angular velocity. It follows that

$$\boldsymbol{\tau}_{\mathrm{net}} = \frac{\mathrm{d}\mathbf{L}}{\mathrm{d}t} = \frac{\mathrm{d}(I\boldsymbol{\omega})}{\mathrm{d}t} = I\frac{\mathrm{d}(\boldsymbol{\omega})}{\mathrm{d}t} = I\boldsymbol{\alpha},$$

where **α** is the angular acceleration of the body, measured in rad s^{-2}.

Proof of the equivalence of definitions

The definition of angular momentum for a single particle is:

$$\mathbf{L} = \mathbf{r} \times \mathbf{p}$$

where "×" indicates the vector cross product and **p** is the particle's linear momentum. The time-derivative of this is:

$$\frac{d\mathbf{L}}{dt} = \mathbf{r} \times \frac{d\mathbf{p}}{dt} + \frac{d\mathbf{r}}{dt} \times \mathbf{p}.$$

This result can easily be proven by splitting the vectors into components and applying the product rule. Now using the definitions of velocity $\mathbf{v} = d\mathbf{r}/dt$, acceleration $\mathbf{a} = d\mathbf{v}/dt$ and linear momentum $\mathbf{p} = m\mathbf{v}$,

$$\frac{d\mathbf{L}}{dt} = \mathbf{r} \times m\frac{d\mathbf{v}}{dt} + \mathbf{v} \times m\mathbf{v}.$$

The cross product of any vector with itself is zero, so the second term vanishes. Hence with the definition of force $\mathbf{F} = m\mathbf{a}$ (Newton's 2nd law),

$$\frac{d\mathbf{L}}{dt} = \mathbf{r} \times \mathbf{F}.$$

Then by definition, torque $\boldsymbol{\tau} = \mathbf{r} \times \mathbf{F}$.

If multiple forces are applied, Newton's second law instead reads $\mathbf{F}_{\mathrm{net}} = m\mathbf{a}$, and it follows that

$$\frac{d\mathbf{L}}{dt} = \mathbf{r} \times \mathbf{F}_{\mathrm{net}} = \boldsymbol{\tau}_{\mathrm{net}}.$$

The proof relies on the assumption that mass is constant; this is valid only in non-relativistic systems in which no mass is being ejected.

Units

Torque has dimensions of force times distance. Official SI literature suggests using the unit newton meter (N m) or joule per radian.[7] The unit newton meter is properly denoted "N m" or "N·m", but not other combinations[8] (this avoids ambiguity—for example, mN is the symbol for millinewtons, nm is the symbol for nanometers, etc.)

The joule, which is the SI unit for energy or work, is dimensionally equivalent to a N m, but this unit is

not used for torque. Energy and torque are entirely different concepts, so the practice of using different unit names for them helps avoid mistakes and misunderstandings.[7] The dimensional equivalence of these units, of course, is not simply a coincidence: a torque of 1 N m applied through a full revolution will require an energy of exactly 2π joules. Mathematically,

$$E = \tau\theta$$

where E is the energy, τ is magnitude of the torque, and θ is the angle moved (in radians). This equation motivates the alternate unit name of "joules per radian".[7]

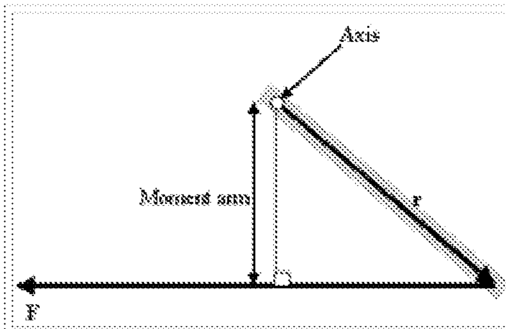
Other non-SI units of torque include "pound-force-feet" or "foot-pounds-force" or "inch-pounds-force" or "ounce-force-inches" or "meter-kilograms-force" or "kilogrammeter" (kgm). For all these units, the word "force" is often left out,[9] for example abbreviating "pound-force-foot" to simply "pound-foot". (In this case, it would be implicit that the "pound" is pound-force and not pound-mass.)

Special cases and other facts

Moment arm formula

A very useful special case, often given as the definition of torque in fields other than physics, is as follows:

$$|\tau| = (\text{moment arm})(\text{force}).$$



Moment arm diagram

The construction of the "moment arm" is shown in the figure below, along with the vectors **r** and **F** mentioned above. The problem with this definition is that it does not give the direction of the torque but only the magnitude, and hence it is difficult to use in three-dimensional cases. If the force is perpendicular to the displacement vector **r**, the moment arm will be equal to the distance to the centre, and torque will be a maximum for the given force. The equation for the magnitude of a torque, arising from a perpendicular force:

$$|\tau| = (\text{distance to center})(\text{force}).$$

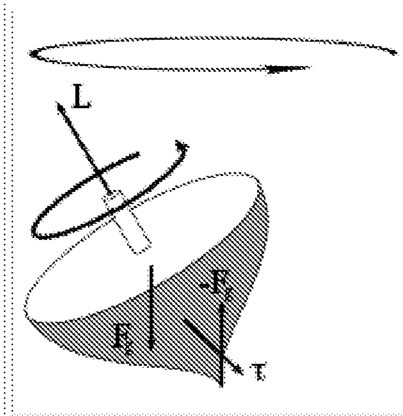
For example, if a person places a force of 10 N on a spanner (wrench) which is 0.5 m long, the torque will be 5 N m, assuming that the person pulls the spanner by applying force perpendicular to the spanner.

Static equilibrium

For an object to be in static equilibrium, not only must the sum of the forces be zero, but also the sum of the torques (moments) about any point. For a two-dimensional situation with horizontal and vertical forces, the sum of the forces requirement is two equations: $\Sigma H = 0$ and $\Sigma V = 0$, and the torque a third equation: $\Sigma\tau = 0$. That is, to solve statically determinate equilibrium problems in two-dimensions, we use three equations.

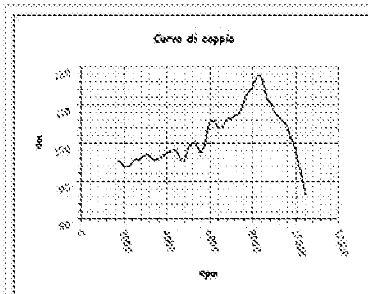
Net Force vs. Torque

When the net force on the system is zero, the torque measured from any point in space is the same. For example, the torque on a current-carrying loop in a uniform magnetic field is the same regardless of your point of reference.



The torque caused by the two opposing forces $\mathbf{F}_g$ and $-\mathbf{F}_g$ causes a change in the angular momentum $\mathbf{L}$ in the direction of that torque. This causes the top to precess.

Machine torque



Torque curve of a motorcycle ("BMW K 1200 R 2005"). The horizontal axis is the speed (in rpm) that the wheels are turning, and the vertical axis is the torque (in Newton metres) that the engine is capable of providing at that speed.

Torque is part of the basic specification of an engine: the power output of an engine is expressed as its torque multiplied by its rotational speed of the axis. Internal-combustion engines produce useful torque only over a limited range of rotational speeds (typically from around 1,000–6,000 rpm for a small car). The varying torque output over that range can be measured with a dynamometer, and shown as a torque curve. The peak of that torque curve occurs somewhat below the overall power peak. The torque peak cannot, by definition, appear at higher rpm than the power peak.

Understanding the relationship between torque, power and engine speed is vital in automotive engineering, concerned as it is with transmitting power from the engine through the drive train to the wheels. Power is a function of torque and engine speed. The gearing of the drive train must be chosen appropriately to make the most of the motor's torque characteristics. Power at the drive wheels is equal to engine power less mechanical losses regardless of any gearing between the engine and drive wheels.

Steam engines and electric motors tend to produce maximum torque close to zero rpm, with the torque diminishing as rotational speed rises (due to increasing friction and other constraints). Reciprocating steam engines can start heavy loads from zero RPM without a clutch.

Relationship between torque, power and energy

If a force is allowed to act through a distance, it is doing mechanical work. Similarly, if torque is allowed to act through a rotational distance, it is doing work. Mathematically, for rotation about a fixed axis through the center of mass,

$$W = \int_{\theta_1}^{\theta_2} \tau \ \mathrm{d}\theta,$$

where W is work, τ is torque, and θ_1 and θ_2 represent (respectively) the initial and final angular positions of the body.[10] It follows from the work-energy theorem that W also represents the change in the rotational kinetic energy K_{rot} of the body, given by

$$K_{\mathrm{rot}} = \tfrac{1}{2} I \omega^2,$$

where I is the moment of inertia of the body and ω is its angular speed.[10]

Power is the work per unit time, given by

$$P = \boldsymbol{\tau} \cdot \boldsymbol{\omega},$$

where P is power, $\boldsymbol{\tau}$ is torque, $\boldsymbol{\omega}$ is the angular velocity, and $\cdot$ represents the scalar product.

Mathematically, the equation may be rearranged to compute torque for a given power output. Note that the power injected by the torque depends only on the instantaneous angular speed - not on whether the angular speed increases, decreases, or remains constant while the torque is being applied (this is equivalent to the linear case where the power injected by a force depends only on the instantaneous speed - not on the resulting acceleration, if any).

In practice, this relationship can be observed in power stations which are connected to a large electrical power grid. In such an arrangement, the generator's angular speed is fixed by the grid's frequency, and the power output of the plant is determined by the torque applied to the generator's axis of rotation.

Consistent units must be used. For metric SI units power is watts, torque is newton meters and angular speed is radians per second (not rpm and not revolutions per second).

Also, the unit newton meter is dimensionally equivalent to the joule, which is the unit of energy. However, in the case of torque, the unit is assigned to a vector, whereas for energy, it is assigned to a scalar.

Conversion to other units

For different units of power, torque, or angular speed, a conversion factor must be inserted into the equation. Also, if rotational speed (revolutions per time) is used in place of angular speed (radians per time), a conversion factor of 2π must be added because there are 2π radians in a revolution:

$$\mathrm{power} = \mathrm{torque} \times 2\pi \times \mathrm{rotational\ speed},$$

where rotational speed is in revolutions per unit time.

Useful formula in SI units:

$$\text{power (kW)} = \frac{\text{torque (N} \cdot \text{m)} \times 2\pi \times \text{rotational speed (rpm)}}{60000}$$

where 60,000 comes from 60 seconds per minute times 1000 watts per kilowatt.

Some people (e.g. American automotive engineers) use horsepower (imperial mechanical) for power, foot-pounds (lbf·ft) for torque and rpm (revolutions per minute) for angular speed. This results in the formula changing to:

$$\text{power (hp)} = \frac{\text{torque(lbf} \cdot \text{ft)} \times 2\pi \times \text{rotational speed (rpm)}}{33000}.$$

The constant below in, ft·lbf./min, changes with the definition of the horsepower; for example, using metric horsepower, it becomes ~32,550.

Use of other units (e.g. BTU/h for power) would require a different custom conversion factor.

Derivation

For a rotating object, the *linear distance* covered at the circumference in a radian of rotation is the product of the radius with the angular speed. That is: linear speed = radius × angular speed. By definition, linear distance=linear speed × time=radius × angular speed × time.

By the definition of torque: torque=force × radius. We can rearrange this to determine force=torque ÷ radius. These two values can be substituted into the definition of power:

$$\text{power} = \frac{\text{force} \times \text{linear distance}}{\text{time}} = \frac{\left(\frac{\text{torque}}{r}\right) \times (r \times \text{angular speed} \times t)}{t} =$$

The radius r and time t have dropped out of the equation. However angular speed must be in radians, by the assumed direct relationship between linear speed and angular speed at the beginning of the derivation. If the rotational speed is measured in revolutions per unit of time, the linear speed and distance are increased proportionately by 2π in the above derivation to give:

$$\text{power} = \text{torque} \times 2\pi \times \text{rotational speed}.$$

If torque is in lbf·ft and rotational speed in revolutions per minute, the above equation gives power in ft·lbf/min. The horsepower form of the equation is then derived by applying the conversion factor 33000 ft·lbf/min per horsepower:

$$\text{power} = \text{torque} \times 2\pi \times \text{rotational speed} \cdot \frac{\text{ft} \cdot \text{lbf}}{\text{min}} \times \frac{\text{horsepower}}{33000 \cdot \frac{\text{ft} \cdot \text{lbf}}{\text{min}}} \approx \frac{\text{torque}}{52}$$

because $5252.113122... = \frac{33000}{2\pi}$.

Principle of Moments

The Principle of Moments, also known as Varignon's theorem (not to be confused with the geometrical theorem of the same name) states that the sum of torques due to several forces applied to *a single* point is equal to the torque due to the sum (resultant) of the forces. Mathematically, this follows from:

$$(\mathbf{r} \times \mathbf{F}_1) + (\mathbf{r} \times \mathbf{F}_2) + \cdots = \mathbf{r} \times (\mathbf{F}_1 + \mathbf{F}_2 + \cdots).$$

See also

- Conversion of units
- Angular momentum
- Mechanical equilibrium
- Moment (physics)
- Rigid body dynamics
- Statics
- Torque converter
- Torque screwdriver
- Torque limiter
- Torque wrench
- Torque tester
- Torsion (mechanics)
- Couple (mechanics)

References

1. ^ Serway, R. A. and Jewett, Jr. J. W. (2003). *Physics for Scientists and Engineers*. 6th Ed. Brooks Cole. ISBN 0-53440-842-7.
2. ^ *Physics for Engineering* by Hendricks, Subramony, and Van Blerk, page 148, Web link (http://books.google.com/books?id=8Kp-UwV4o0gC&pg=PA148)
3. ^ [a] [b] [c] *Dynamics, Theory and Applications* by T.R. Kane and D.A. Levinson, 1985, pp. 90-99: Free download (http://ecommons.library.cornell.edu/handle/1813/638)
4. ^ Tipler, Paul (2004). *Physics for Scientists and Engineers: Mechanics, Oscillations and Waves, Thermodynamics (5th ed.)*. W. H. Freeman. ISBN 0-7167-0809-4.
5. ^ "Right Hand Rule for Torque" (http://hyperphysics.phy-astr.gsu.edu/hbase/tord.html). http://hyperphysics.phy-astr.gsu.edu/hbase/tord.html. Retrieved 2007-09-08.
6. ^ [a] [b] Halliday, David; Resnick, Robert (1970). *Fundamentals of Physics*. John Wiley & Sons, Inc.. p. 184–85.
7. ^ [a] [b] [c] From the official SI website (http://www.bipm.org/en/si/si_brochure/chapter2/2-2/2-2-2.html): "...For example, the quantity torque may be thought of as the cross product of force and distance, suggesting the unit newton metre, or it may be thought of as energy per angle, suggesting the unit joule per radian."
8. ^ "SI brochure Ed. 8, Section 5.1" (http://www1.bipm.org/en/si/si_brochure/chapter5/5-1.html). Bureau International des Poids et Mesures. 2006. http://www1.bipm.org/en/si/si_brochure/chapter5/5-1.html. Retrieved 2007-04-01.
9. ^ See, for example: "CNC Cookbook: Dictionary: N-Code to PWM" (http://www.cnccookbook.com/MTCNCDictNtoPWM.htm). http://www.cnccookbook.com/MTCNCDictNtoPWM.htm. Retrieved 2008-12-17.
10. ^ [a] [b] Kleppner, Daniel; Kolenkow, Robert (1973). *An Introduction to Mechanics*. McGraw-Hill. p. 267–68.

External links

- Power and Torque Explained (http://www.epi-eng.com/ET-PwrTrq.htm) A clear explanation of the relationship between Power and Torque, and how they relate to engine performance.
- "Horsepower and Torque" (http://craig.backfire.ca/pages/autos/horsepower) An article showing how power, torque, and gearing affect a vehicle's performance.
- "Torque vs. Horsepower: Yet Another Argument" (http://kevinthenerd.googlepages.com/torque_vs_hp.html) An automotive perspective
- a discussion of torque and angular momentum in an online textbook (http://www.lightandmatter.com/html_books/2cl/ch05/ch05.html)
- *Torque and Angular Momentum in Circular Motion* (http://www.physnet.org/modules/pdf_modules/m34.pdf) on Project PHYSNET (http://www.physnet.org/).
- An interactive simulation of torque (http://www.phy.hk/wiki/englishhtm/Torque.htm)
- Torque Unit Converter (http://www.lorenz-messtechnik.de/english/company/torque_unit_calculation.php)
- www.mechanismen.be-what is a moment (dutch) (http://www.mechanismen.be/theoretische_mechanica/momenten/momenten-theorie-1.htm)

Retrieved from "http://en.wikipedia.org/wiki/Torque"
Categories: Fundamental physics concepts | Engine technology | Physical quantities | Introductory physics | Rotation

---